Efficient CAD of a Class of Waveguide Discontinuities via an Hybrid Finite-Element/Mode-Matching and Modified Transverse-Resonance Analysis

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Abstract— **We introduce a new technique for the efficient CAD of a class of waveguide components. The basic idea is to apply the Finite Element Method / Mode–Matching (FEM/MM) technique in a direction different from the propagation one, by using a Modified Transverse Resonance (MTR) approach. In this way it is unnecessary to use a fully three-dimensional numerical electromagnetic simulator and several frequency points are computed by just one analysis.**

I. Introduction

Efficient analysis of waveguide components is required for CAD involving optimization and tolerance analysis. The most efficient methods are those based on significant analytical pre–processing, such as e.g. modal analysis techniques. However the latter techniques suffer from being limited only to waveguides with separable cross–sections, as shown for example in Fig. 1a.

In order to circumvent such a limitation and to extend their range of applicability, a hybrid FEM (Finite Element Method) / MM (Mode–Matching) technique has been studied recently [1], [2]. In this approach a two-dimensional FEM (2D-FEM) is used for computing the modes of the non–separable cross–sections and the MM is then applied for scattering matrix computation at the junction between the two different waveguides, as illustrated in Fig. 1b.

However, several structures of practical interest as that represented in Fig. 1c, are of difficult solution by the above approaches. On the other hand this class of structures present a certain symmetry along the y–direction that makes a fully three–dimensional (3D) computation unnecessary. In fact, by using a Transverse Resonance (TR) approach [3], we can place two metallic walls along the longitudinal direction z. In this way we obtain a closed resonator; the latter can now be analyzed, in a very efficient manner, by an application of the above hybrid FEM/MM technique but applied along the *transverse* y-direction. This approach significantly enlarge the class of waveguide discontinuities that can be analyzed by the hybrid FEM/MM technique by making use of the geometrical invariances present in the structure.

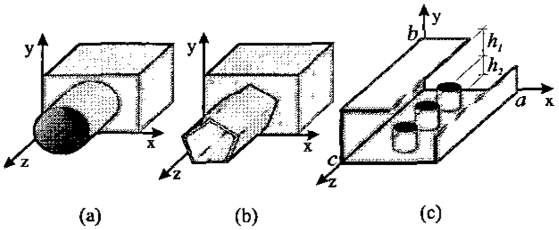

Fig. 1. A junction between two waveguides of separable cross–sections (a) is efficiently analyzed by the MM (Mode-Matching) technique; when one (or both) waveguides are of non–separable cross–section (b) the hybrid FEM/MM method can be applied. For structures like those in (c), it is possible to apply the MTR-FEM/MM, i.e. to perform the MM in a direction different from the propagation one.

Classical TR method requires, for a single frequency point, to find three sets of short–circuit plane positions that share the same resonant frequency. These positions are obtained by an iterative search, hence a large number of analysis are needed. In our case this approach would be very time consuming since for each position of the short-circuit planes a new set of modes would have to be computed. We therefore introduce a Modified Transverse Resonance (MTR) which, based on the knowledge of the resonant field inside the cavity, allows to obtain, for each short–circuit plane position, the response over several frequency points.

In the next section we illustrate the method with the example of the structure shown in Fig. 1c.

II. Method description

The proposed method is based on the finite-element/mode-matching (FEM/MM) analysis and on a Modified form of the Transverse Resonance (MTR) for evaluating the frequency response of the device under analysis from its natural resonances.

A. Finite element method / mode–matching

The technique follows the standard approach proposed e.g. [1], [2]. The noticeable difference is that, in the present approach, the modal technique is ap-

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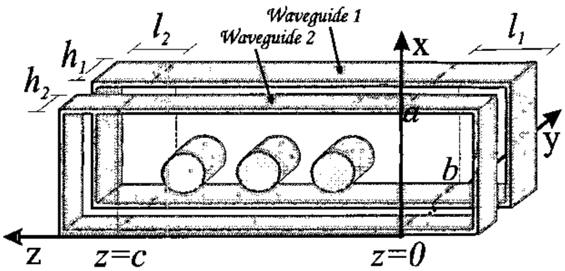

Fig. 2. Example of the structure to be simulated. The direction of propagation is along z axis, but we analyze such a structure with a mode-matching along y axis. The modes in waveguide 1 and 2 are computed by a 2D-FEM.

plied in the transverse y-direction. As an example, let us analyze the structure in Fig. 1c, which consists of a rectangular waveguide containing three circular posts of the same height and possibly different diameters. Input and output ports are at $z = 0$ and $z = c$; in order to apply the proposed technique we insert an additional waveguide length at both sides of the structure and we terminate it by metallic walls, hence obtaining a closed metallic resonator. With reference to Fig. 2, such a resonator can be seen, in the transverse y-direction, as a waveguide step in which ports at $y = 0$ and $y = b$ are short-circuited. In order to avoid a time-consuming 3D analysis, we note that this structure is formed by the junctions of two different waveguides. In fact, by looking at the structure in the y-direction, we have for $h_2 < y < b$ an empty rectangular waveguide and for $0 < y < h_2$ a rectangular waveguide with three circular inner conductors. The scattering matrix of the waveguide step can be evaluated by using the mode-matching technique and, as the waveguide cross-sections are non-separable, the waveguide modes are evaluated by using the 2D-FEM.

Once the scattering matrix of the waveguide step in Fig. 2 has been computed, the resonator eigenfrequencies and eigenmodes are obtained by solving an eigenvalue problem:

$$\left(\begin{bmatrix} [S11] & [S12] \\ [S21] & [S22] \end{bmatrix} - \lambda[I]\right)\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} = 0 \qquad (1)$$

where $[Sij]$ represents the scattering sub-matrix relating modes of waveguide i and waveguide j, $[I]$ represents the identity matrix, while λ and $\left[\mathbf{a}_1^T, \mathbf{a}_2^T\right]$ represent eigenvalue and eigenvector, respectively.

The eigenfrequency of the resonator are those for which $\lambda = -1$. In fact, for such a value of λ, the eq. (1) becomes:

$$-\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} = \begin{bmatrix} [S11] & [S12] \\ [S21] & [S22] \end{bmatrix}\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} \qquad (2)$$

and the condition of the short circuit (requiring incident and reflected wave of the same amplitude and

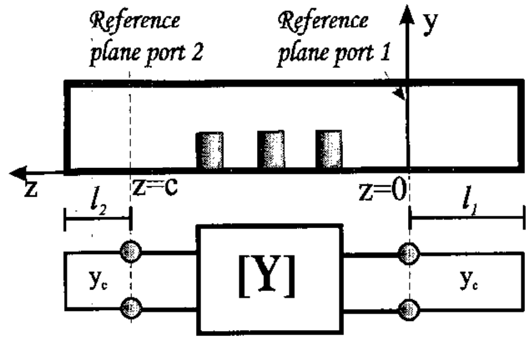

Fig. 3. The device and its equivalent network (admittance representation). Both device and equivalent circuit are short-circuited at the distance l_1 and l_2 from port 1 and 2, respectively, hence providing a metallic resonator.

different sign) are satisfied. From the knowledge of incident ($= \left[\mathbf{a}_1^T, \mathbf{a}_2^T\right]$) and reflected ($= -\left[\mathbf{a}_1^T, \mathbf{a}_2^T\right]$) waves, it is also possible to obtain the resonant electromagnetic field as waveguide modal expansion.

Finally, from the resonant modes we are able to extract the scattering parameters as shown in sec. II-B.

B. Modified transverse resonance analysis

Under appropriate conditions, the knowledge of the resonant modes of a structure allows a simple and fast evaluation of the admittance parameters. As an example, let us consider our 3-posts in waveguide and consider its equivalent network as illustrated in Fig. 3. Let us suppose that the branch of length l_i attached to the reference plane i is an homogeneous waveguide working in a frequency range where only the fundamental mode is propagating. Let us also suppose that only the fundamental mode is present at the reference planes while other modes are negligible.

Under these conditions, the device can be represented by an admittance matrix relative only to the fundamental modes; the short-circuited homogeneous waveguides of length l_i are represented by short-circuited transmission lines of characteristic impedance y_c.

Supposing the device to be reciprocal ($Y_{12} = Y_{21}$) and symmetric ($Y_{11} = Y_{22}$), the electromagnetic field on the reference planes satisfies the relation:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{11} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1 & V_2 \\ V_2 & V_1 \end{bmatrix}\begin{bmatrix} Y_{11} \\ Y_{12} \end{bmatrix} \qquad (3)$$

where, I_i and V_i represent the modal voltages and currents of the i-th port fundamental mode, respectively, while Y_{ij} are the admittance matrix elements. Due to the presence of the short circuits at the input and output ports, the relation between modal voltages and currents on the reference planes is:

$$I_1 = -jy_c \cot(\beta l_1) V_1 \qquad (4)$$
$$I_2 = jy_c \cot(\beta l_2) V_2$$

where y_c and β represent the input (output) port characteristic admittance and propagation constant, respectively. In the reference planes, the electromagnetic field distribution of the selected resonant modes can be written in terms of the input (output) port fundamental modes only, providing the modal voltages V_1 and V_2. The modal voltages extracted from the transverse modes satisfy (3) and (4). By inverting system (3) and by using equation (4), the admittance elements can be found in terms of the extracted modal voltages as:

$$\begin{bmatrix} Y_{11} \\ Y_{12} \end{bmatrix} = \frac{1}{V_1^2 - V_2^2} \begin{bmatrix} V_1 & -V_2 \\ -V_2 & V_1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{jy_c}{V_1^2 - V_2^2} \begin{bmatrix} V_1^2 & -V_2^2 \\ -V_2 V_1 & V_1 V_2 \end{bmatrix} \begin{bmatrix} -\cot(\beta l_1) \\ \cot(\beta l_2) \end{bmatrix} \qquad (5)$$

Note that the (3) can not be inverted if $V_1 = V_2$. Hence, in order to obtain different values of V_1 and V_2, the distance l_1 and l_2 must be different.

From the admittance matrix the scattering parameters are derived in the standard way.

In the following we briefly summarize the procedure (see Fig. 3 for reference).

B.1 A summary of the procedure

1. Insert short–circuiting planes at distances l_1 and l_2 away from the reference planes 1 and 2.
2. Evaluate the resonant modes of the entire device, as explained in sec. II-A.
3. Evaluate the characteristic admittance y_c and the propagation constant β of the input/output waveguides at the resonant frequencies.
4. Use the extracted modal voltages on the reference planes 1 and 2 to find out, by using the system (5), the admittance matrix elements.

Whenever the number of frequency points is not sufficient, we can repeat the whole procedure by performing another FEM/MM analysis by slightly changing l_1 or l_2 or both of them.

III. Results

The novelty of the suggested approach consists in applying the MTR procedure to the FEM/MM analysis so as to extend its range of applicability. Since the application of the FEM/MM has been already discussed [1], [2], we first present, in the subsection A, two examples that don't require the MM procedure and then, in the subsection B, an example that require the entire procedure (MTR-FEM/MM).

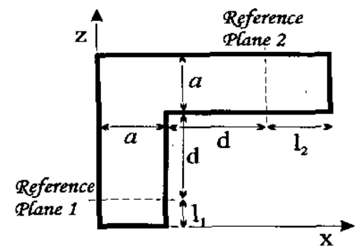

Fig. 4. H-Plane right angle corner bend. Dimensions (mm): a=7.1 d=7 l_1=3 l_2=5.

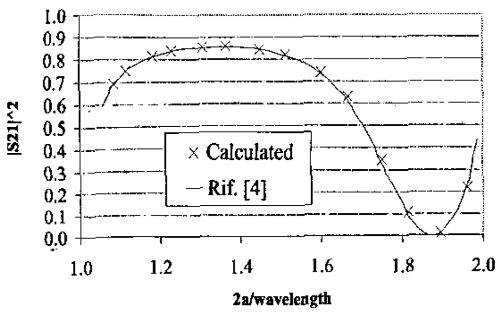

Fig. 5. Transmission coefficient of the bend of Fig. 4.

A. *Application of the MTR-FEM*

A.1 Example 1: H-plane rectangular waveguide bend

The first example consists in the H-plane right angle corner bend showed in Fig. 4. As stated in step 1 of the procedure, shorting planes at distances l_1 and l_2 away from the reference planes 1 and 2 have been inserted. Then a 2D-FEM analysis of the cross–section to derive the resonant modes has been performed and finally MTR has been applied. In Fig. 5 our results have been compared with those obtained in [4], showing an excellent agreement. The bend cross–section has been meshed by using 3000 triangles and only one 2D-FEM analysis was necessary to obtain the frequency points reported in Fig. 5.

A.2 Example 2: H-plane three–cavity filter

As a second example, the three-cavity filter in WR-75 waveguide presented in [5] has been analyzed. Filter shape and dimensions are reported in Fig. 6. Note that filter cavities are not rectangular as its corners have a certain roundness, so such a filter could not be analyzed using the classical mode-matching in the propagation direction. In Fig. 7 the scattering parameters obtained with our method have been compared with those obtained in [5] showing an excellent agreement. Six 2D-FEM analyses has been performed in order to obtain the frequency points reported in Fig. 7. The filter cross-section has been

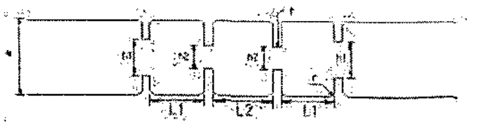

Fig. 6. WR75 Filter Geometry. Dimensions (in mm): a=19.05; t=2.0; r=1.5, h1=9.194; h2=5.858; L1=13.617; L2=14.966.

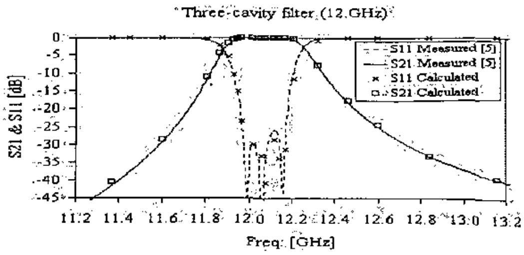

Fig. 7. Scattering parameters of the filter of Fig. 6. Comparison between simulated and measured data.

meshed with 3000 triangles.

B. *Application of the MTR-FEM/MM*

As one application of the proposed method we have considered e.g. the analysis of a single metallic post discontinuity as shown in Fig. 8.

B.1 Circular post

In Fig. 9 the analysis of the circular metallic post of Fig. 8a, obtained by the proposed method, has been compared with those obtained by a classical 3D-FEM by using the commercial software tool HFSS by Ansoft.

B.2 Rectangular post

In order to ascertain the accuracy of the proposed method, as a test structure, the rectangular post of Fig. 8b has been investigated. Such a structure can be analyzed along both the longitudinal (z axis) and transverse (y axis) direction by using FEM/MM and MTR-FEM/MM, respectively. The comparison of the transmission coefficient amplitude (dB) is presented in Tab. I.

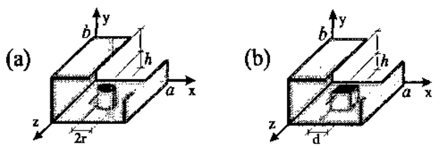

Fig. 8. Circular (a) and squared (b) post in rectangular waveguide. Dimensions (mm): a=80, b=40, h=30.6, r=5.64, d=10.

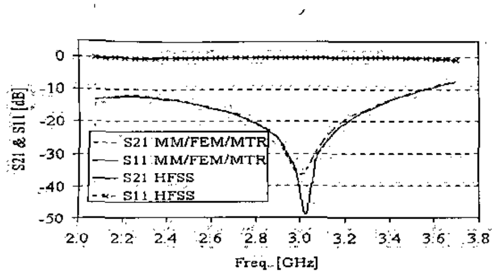

Fig. 9. Scattering parameters of a circular post in rectangular waveguide. Comparison between our data and those obtained by a classical 3D-FEM (HFSS).

Freq. (GHz)	FEM/MM	MTR-FEM/MM
2.0	12.8	13.2
2.4	12.8	13.1
2.9	20.6	20.9
3.4	14.9	15.0

TABLE I

Transmission coefficient amplitude *(dB)* of a rectangular waveguide with a rectangular post inside. Comparison between FEM/MM and MTR-FEM/MM analysis.

IV. Conclusion

The Finite-Element / Mode-Matching (FE/MM) Method has extended significantly the applicability of the modal techniques but, with the standard approach, it can only be applied when the discontinuities are along the direction of propagation.

In this paper, by introducing the Modified Transverse Resonance (MTR) technique, we have extended the FEM/MM applicability also to the case of discontinuities in the transverse direction. Such extension allows to analyze, in an efficient manner, a class of waveguide components, as, for example, filters with posts.

Comparisons with reference data have shown the validity of the proposed approach.

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